The Battle Between Impeccable Intonation and Maximized Modulation

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Abstract

Equal temperament represents a way of completing the musical circle, and systematically compensating for the Pythagorean comma. Pythagoras discovered this acoustical problem around 550 B.C., and since that time music theorists have debated how to deal with it. The problem is that no perfect solution exists—something must be compromised. As musical styles developed, specific factors and harmonic tendencies led to the gradual adoption of equal temperament. Early in music history, theorists preferred systems which kept acoustical purity relatively intact. Pythagorean intonation and just intonation serve as two examples. However, the move from modality to tonality decentralized the melody as the dominating feature of a composition. Correspondingly, this raised the importance of harmonic structure, and introduced the idea of modulation. Not all tuning systems allow a performer to easily change keys; most systems contain some type of wolf fifth. This interval sounds exceedingly dissonant, due to its distance from an ideal frequency ratio. Thus, composers had to avoid certain keys, like F# Major, or Bb Minor. During this time, hundreds of different meantone temperaments arose, all of which deal with the Pythagorean comma in slightly different ways. These temperaments attempt to balance pure acoustics and modulatory freedom. Eventually, as chromaticism became increasingly common, so did equal temperament. Musicians traded true intonation for the ability to play in any key at any time. While equal temperament is now universally hailed as the standard tuning system, it is not perfect. Rather, it represents a compromise designed to best accommodate the needs of tonal music since the Baroque Era. I will mathematically show the problems encountered when creating a tuning system, and discuss the various known solutions. I will then use historical documentation to show how musicians eventually landed on equal temperament as the most complete solution.

Keywords

Equal Temperament, Mean Tone Temperament, Pythagorean Intonation, Just Intonation, Harmonics, Music Theory

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The Battle Between Impeccable Intonation and Maximized Modulation

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Equal temperament represents a way of completing the musical circle and systematically compensating for the Pythagorean comma—a fundamental inconsistency in harmony and tuning. Pythagoras discovered this acoustical problem around 550 B.C. Since that time, music theorists have debated how to deal with it. The acoustical problem is that 12 perfect fifths and 7 octaves are different intervals. Unfortunately, no perfect solution exists to this problem— something must be compromised. Four of the major compromises are Pythagorean intonation, just intonation, equal temperament, and meantone temperament. However, understanding these systems requires a basic knowledge of acoustics and harmony. Throughout the course of history, musicians used the tuning or temperament that made their own music sound best. Eventually, they traded true intonation for the ability to play in any key at any time. While equal temperament is now universally hailed as the standard tuning system, it is not perfect. Rather, it represents a compromise designed to best accommodate the needs of tonal music since the Baroque era.

What is temperament? To answer this question, one must first understand the basics of musical harmony. Scientifically, a single note, or pitch, represents a sound wave of a specific frequency.[1] Each note corresponds to a particular frequency. For instance, many orchestras tune to the frequency of 440 Hz. Frequency measures the number of vibrations per second; thus, a frequency of 440 Hz means that the sound waves move 440 times every second. The higher the frequency, the higher the note— the lower the frequency, the lower the note. When two different pitches are played simultaneously, the frequency relationship between the notes

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[1] John Fauvel, Raymond Flood, and Robin J. Wilson, *Music and Mathematics: From Pythagoras to Fractals* (New York: Oxford University Press, 2003), 13.

determines how consonant or dissonant they sound together. As noted by Pythagoras, the most consonant sounds are generated when the frequencies of the notes can be expressed as a simple ratio (i.e. 2:1, 3:2, 4:3, etc.).[2] Two notes sound pleasant together if their frequencies can be written as integer ratios of each other. What does this mean? Suppose the note A4 (440 Hz) is played. The frequency of the most consonant note would be either 220 Hz (2:1 below) or 880 Hz (2:1 above). Musically, this 2:1 ratio corresponds to an octave; thus, A3 has a frequency of 220 Hz, A2 has a frequency of 110 Hz, and so on. The next simplest ratio comes from the next lowest pair of positive integers—3:2. When the two frequencies 440 Hz and 660 Hz (3 ÷ 2 × 440 = 660) are sounded together, the result is consonant. Musically, this ratio corresponds to a perfect fifth. Other simple ratios can be used to produce the perfect fourth (4:3), major third (5:4), minor third (6:5), and major sixth (5:3).

Figure 1: Chart of Different Frequency Ratios.[3]

Interval	Frequency Ratio	Decimal	Cents
Perfect Octave	2:1	2.00	1200
Perfect Fifth	3:2	1.50	702
Perfect Fourth	4:3	1.33…	498
Major Third	5:4	1.25	386
Minor Third	6:5	1.20	316
Major Sixth	5:3	1.66…	884

Figure 1 provides a succinct summary of the basic consonant frequency relationships. The final column expresses the frequency ratios in a slightly different way, using cents.[4]

Cent value = 3986 × log (frequency ratio)

Note that here, log (frequency ratio) represents the base 10 logarithm of the frequency ratio. Thus, an octave has 1200 cents because 3986 × log (2) = 3986 × 0.301 = 1200. Similar calculations can be done for the other ratios. While this process may seem complex, it can be done rather quickly and easily using a calculator. To summarize, cent values are just a more precise way of indicating how far apart two notes are. Cent values

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[2] Fauvel, Flood, and Wilson, *Music and Mathematics*, 13.
[3] Thomas Donahue, *A Guide to Musical Temperament* (Lanham, Maryland: The Scarecrow Press, 2005), 5.
[4] Ibid.

are also convenient because of mathematical logarithm rules. For instance, what is the distance between A4 and E5? We must multiply frequency ratios: A4 to A5 to E5. 2/1 x 3/2 = 3/1. However, cent values can be added together. Going from A4 to A5 to E5, add an octave and a fifth. 1200 + 702 = 1902. Thus, the ability to add cent values is quite helpful. The greater the number of cents, the greater the distance between two notes.

With this understanding, a problem arises. If a person begins on the lowest C of the piano (C1) and ascends 7 perfect octaves, he/she will land on C8. Similarly, that person could also begin on C1 and ascend 12 perfect fifths, landing on C8. However, when doing this process mathematically, the resulting frequencies are different. As shown in Figure 1, an octave is equivalent to 1200 cents. Seven octaves then correspond to a frequency change of $7 \times 1200 = 8400$ cents. However, marching up twelve perfect fifths, there is a corresponding frequency change of $12 \times 702 = 8424$ cents. Thus, there is a 24-cent difference between this chain of octaves and fifths. The 24-cent difference can also be described mathematically by the frequency ratio $3^{12}/2^{19}$. This difference has been known since the time of Pythagoras in 550 B.C.[5] Because of this 24-cent difference, known as the Pythagorean comma, the musical "circle" cannot be completed. Note that a 24-cent difference corresponds to approximately 1/3 the difference of a semitone. While it may not seem like much, this difference causes a terrible tuning issue. As Stuart Isacoff put it, "In order for the twelve pitches generated through the proportion 3:2 to complete a path from 'do' to 'do,' the circle has somehow to be adjusted or 'rounded off.'"[6] The problem of the Pythagorean comma is solved using some type of systematic adjustment.

The adjustments can be divided into two general categories: tuning and temperament. According to J. Murray Barbour, a tuning system is one "in which all intervals may be expressed as the ratio of two integers."[7] Conversely, Barbour says that "a temperament is a modification of tuning which needs radical numbers to express the ratios of some or all of its intervals." Radical numbers are those such as $\sqrt{2}$, π, or $5^{1/4}$. They cannot be written as the ratio of two integers. Pythagorean intonation and

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[5] Fauvel, Flood, and Wilson, *Music and Mathematics*, 18.
[6] Stuart Isacoff, *Temperament: How Music Became a Battleground for the Great Minds of Western Civilization* (New York: Vintage, 2003), 65.
[7] J. Murray Barbour, *Tuning and Temperament: A Historical Survey* (East Lansing: Michigan State College Press, 1953), 5.

just intonation represent examples of the former; equal temperament and mean-tone temperament represent examples of the latter.

Throughout the course of music history, hundreds of tuning and temperament systems have been suggested. Several important ones will be summarized here. Pythagorean intonation is a tuning system in which the perfect fifth ratio (3:2) is used to generate the relationship between other notes of a musical scale. In Pythagorean intonation, the major third is represented by a frequency ratio of 81/64 (noticeably different than the 5/4 ratio of the pure major third), and notes such as A♭ and G♯ are not necessarily the same frequency.[8] Just intonation involves simpler ratios for each interval.

Figure 2: Frequency ratios for just intonation.[9]

| Note	C	D	E	F	G	A	B	C'
Frequency	1/1	9/8	5/4	4/3	3/2	5/3	15/8	2/1
Interval	9/8	10/9	16/15	9/8	10/9	9/8	16/15	N/A

The top row of numbers represents the frequency ratio between that note and C. The bottom row of numbers represents the frequency difference between that note and the note to its right. For example, to get from F to G, one must multiply by 9/8: therefore, $4/3 \times 9/8 = 3/2$. To get from F to A, multiply by 9/8 and 10/9. $4/3 \times 9/8 \times 10/9 = 5/3$.

This definition looks very nice on paper and sounds adequate for simpler music. However, to put it bluntly, "the compromise breaks down when one wants to play in another key."[10] See the chart below to understand some of the differences between Pythagorean and just intonation. The noticeable difference between them comes in the definition of the fifth and the resulting wolf fifth.

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[8] Vicente Liern, "On the Construction, Comparison, and Exchangeability of Tuning Systems," *Journal of Mathematics & Music* 9, no. 3 (November 2015): 201, doi:10.1080/17459737.2015.1031468.
[9] Fauvel, Flood, and Wilson, *Music and Mathematics*, 21.
[10] Ibid.

Figure 3: A comparison of just and Pythagorean intonations.[11]
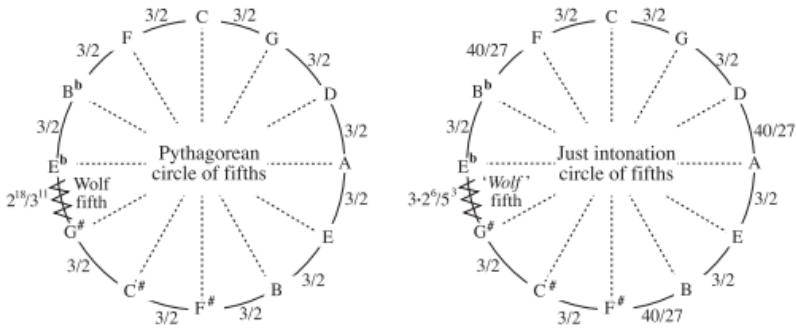
The difference in the wolf fifth is noticeable. The Pythagorean wolf fifth expressed as a decimal is 1.480 or 24-cents flat as discussed before (Pythagorean comma). The just wolf fifth expressed as a decimal is 1.536 or 41-cents sharp.

When any type of chromatic modulation occurs, these issues are essentially irresolvable. Because the ratios between the notes are not consistent, shifting the tonic "do" results in many odd intervals. These can sound very dissonant, and thus systems like just or Pythagorean intonation limit the possibilities of musical performance.

Although its roots are much earlier, equal temperament (ET) is the system that has been widely used and adopted since the mid-eighteenth century.[12] ET first defines the perfect octave to be a 2:1 frequency ratio. Next, the musical scale is broken into twelve notes each equidistant from each other. Each half step, or semitone, corresponds to changing the frequency by a factor of $2^{1/12}$. A whole step indicates a frequency change of $2^{2/12}$ or $2^{1/6}$. After twelve of these equidistant half-step changes, the resulting frequency is $2^{12/12}$ or 2—simply twice the frequency of the original note. Thus, the perfect octave has been reached. ET allows for incredibly easy modulation, because chromatic tones are far less dissonant. While ET may seem like the perfect solution to the problem of the Pythagorean comma, it is not quite that simple. As

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[11] Liern, "On the Construction, Comparison, and Exchangeability of Tuning Systems," 201.

[12] Amanda N. Staufer, "The Unifying Strands: Formalism and Gestalt Theory in the Musical Philosophies of Aristoxenus, Descartes, and Meyer," *Musical Offerings* 9, no. 1 (2018): 34, doi:10.15385/jmo.2018.9.1.3.

mentioned by Ross Duffin, "Nothing can change the fact that the major third of ET is a long way from acoustical purity."[13] L. S. Lloyd says "Musicians know that equal temperament is an acoustical compromise, tolerated by many ears on the piano, and designed to satisfy as completely as possible three incompatible requirements—true intonation, complete freedom of modulation and convenience in practical use in keyed instruments—and that it sacrifices the first of these to the second and third."[14] ET provides complete freedom of modulation as well as practicality and convenience for keyed instruments. However, it does sacrifice true intonation. Consequently, while ET standardizes the distances between pitches, it took time for musicians to accept the aural impurities that ET embraces.

One very early figure to speak on this issue was Aristoxenus, a Greek philosopher who lived about one hundred years after the time of Pythagoras.[15] He asked a very important question—one that is central to the issue of resolving the Pythagorean comma. Which should have priority—aural purity or mathematical perfection? Aristoxenus argued that aural perception should have authority over mathematical ratios. While this idea is a noble conjecture, it does not necessarily present a practical solution. Claudius Ptolemy, a second century mathematician, theorist, and author of the influential book *Harmonics*, believed differently. He disagreed with Aristoxenus, and instead thought that "tuning is best for which ear and ratio are in agreement."[16] A compromise must be reached between rigid mathematical definition and sensory aural perception.

In 1577, Francisco Salinas first mentioned ET as a viable tuning system for certain instruments with fixed pitches.[17] Salinas was an early Spanish theorist whose writings influenced much of Renaissance and Baroque music. He realized that equal temperament was important during the construction of fretted string instruments, particularly the viol.[18]

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[13] Ross W. Duffin, *How Equal Temperament Ruined Harmony (and Why You Should Care)* 1st ed. (New York: W. W. Norton, 2007), 29.

[14] L. S. Lloyd, *Intervals, Scales and Temperaments* (London: Macdonald & Co. Ltd., 1963), 66.

[15] Staufer, "The Unifying Strands," 33.

[16] Barbour, *Tuning and Temperament: A Historical Survey*, 2.

[17] Arthur Daniels, "Microtonality and Mean-Tone Temperament in the Harmonic System of Francisco Salinas," *Journal of Music Theory* 9, no. 1 (1965): 5, doi:10.2307/843148.

[18] Barbour, *Tuning and Temperament: A Historical Survey*, 6.

However, while Salinas conceived the idea theoretically, he did not advocate for it outside the making of the viol. Instead, Salinas supported just intonation which meant that "all intervals are derived from the pure fifth and the pure major third."[19] This contrasted with the Pythagorean system which based small intervals solely off the division of the fifth. Practically, this meant that the Pythagorean system was more complex and less acoustically viable. Salinas's work was instrumental in the development of the music of his time. His tuning system, which used the pure fifth and pure major third, led to the acceptance of the triad as the "basic building block of late Renaissance music."[20] This step had consequences for many years to come.

Although Salinas's just intonation was aurally pleasing, it failed to fully and perfectly tune a keyboard instrument. As mentioned previously in this article, any type of modulation will destroy the sound of just intonation. Harmonies may sound consonant in one key, but after a half-step modulation, all kinds of problems arise. Salinas was aware of this and he knew that he had to compromise somehow when tuning keyboard instruments. According to Arthur Daniels, "Salinas recommends three systems of meantone temperament for keyboard instruments, the first of which was his own invention: the 1/3 comma, 2/7 comma, and the 1/4 comma temperament systems."[21] The second meantone temperament was invented by Gioseffo Zarlino and the third by Pietro Aron.[22] Overall, these three temperaments are constructed using the same process, yet with slightly different specifics.

To understand the details of meantone temperament, one must delve slightly into mathematics. The definition of meantone is more complex than equal temperament. First, some terms must be clarified. In this situation, "comma" refers to a syntonic comma, or the difference between a Pythagorean third (81/64) and a just third (5/4). Written as a ratio, the syntonic comma is 81/80 (81/64 x 80/81 = 5/4). In terms of cent values, the syntonic comma is equal to 21.5 cents (3986 x log [81/80] = 21.5). Meantone temperaments all slightly flatten the fifth by some amount—the three discussed here flatten the fifth by 1/4 comma, 2/7 comma, and 1/3 comma respectively. In 1/4 comma temperament, this is

[19] Daniels, "Microtonality and Mean-Tone Temperament," 5.
[20] Ibid., 6.
[21] Ibid.
[22] Mimi S. Waitzman, "Mean-Tone Temperament in Theory and Practice," *In Theory Only: Journal of the Michigan Music Theory Society* 5, no. 4 (May 1981): 7, 12, https://quod.lib.umich.edu/g/genpub/0641601.

done by equating two ratios: four fifths (x^4) compared to two octaves and a major third (2x2 x 5/4). Basically, this defines the distance from C3 to E5 via fifths (think C3 to G3 to D4 to A4 to E5) to equal the distance from C3 to E5 via two octaves and a third (C3 to C4 to C5 to E5). This definition is shown below mathematically, where x represents the interval of a fifth.

$$x^4 = 2^2(5/4)$$
$$x^4 = 5$$
$$x = 5^{1/4} = 1.49535\ldots$$

Thus, for 1/4 comma meantone temperament, the fifths are not 3/2 = 1.50000 but are instead slightly less. 2/7 comma meantone and 1/3 comma meantone are defined similarly but with the fifth lowered by slightly different amounts. The exact similarities and differences between these temperaments can be seen below.

Figure 4: Comparison of Comma Temperaments.[23]

	Aron's 1/4 Comma Temperament		Zarlino's 2/7 Comma Temperament		Salina's 1/3 Comma Temperament	
Note Name	Distance from Pythagorean Intonation	Cents	Distance from Pythagorean Intonation	Cents	Distance from Pythagorean Intonation	Cents
C	0	0	0	0	0	0
C♯	-7/4	76	-2	70	-7/3	64
D	-1/2	193	-4/7	191	-2/3	190
Eb	-3/4	310	+6/7	313	+1	316
E	-1	386	-8/7	383	-4/3	379
F	+1/4	503	+2/7	504	+1/3	505
F♯	-3/2	579	-12/7	574	-2	569
G	-1/4	697	-2/7	696	-1/3	695
G♯	-2	773	-16/7	817	-8/3	758
A	-3/4	890	-6/7	887	-1	884
Bb	+1/2	1007	+4/7	1008	+2/3	1010
B	-5/4	1083	-10/7	1078	-5/3	1074
C	0	1200	0	1200	0	1200

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[23] Barbour, *Tuning and Temperament: A Historical Survey*, 26.

At first glance, Figure 4 seems quite ambiguous. Taking a closer look, notice that the left column lists the twelve notes (from C to C), and Columns 3, 5, and 7 list the interval size in cents. For example, in Aron's 1/4 comma temperament, C♯ in 76 cents from C, D is 193 cents from C, and so on. Columns 2, 4, and 6 indicate how far that note is from Pythagorean intonation (in terms of a syntonic comma). Notice that in each system, the fifth (C to G) is tempered down by 1/4, 2/7, or 1/3 of a syntonic comma—hence the name of each temperament. Tempering each fifth results in deviations from Pythagorean intonation. Consequently, the error of all other notes can be found.

Zarlino also worked extensively on various tuning systems. A contemporary of Salinas, Zarlino is mentioned alongside Marin Mersenne and Jean-Philippe Rameau as one of "the great music theorists."[24] Barbour makes the interesting point that these three "presented just intonation as the theoretical basis of the scale, but temperament as a practical necessity."[25] This common position demonstrates the dilemma that composers like Zarlino were facing. However, while a perfect answer remained elusive, the temperaments Zarlino used were still regarded as satisfactory. While equal temperament was used for fretted instruments, meantone temperament was used for keyboard instruments.[26] While meantone temperament is rarely used to tune keyboards today, Zarlino considered it "very pleasing for all purposes."[27] Specifically, Zarlino created the 2/7 comma meantone temperament. This system has a few positives, but overall, it is "inferior to the 1/4 comma system."[28] Essentially, the greater amount of tempering (2/7 > 1/4) causes intervals to be less pure. Why use it then? Based on the design of Zarlino's system, the impurities are regular; major and minor thirds and sixths are all 1/7 comma off.[29] This small detail demonstrates why so many meantone temperaments arose; theorists sought to minimize slight discrepancies to get a better sound overall. These temperaments do allow for modest modulation, but unlike ET, they completely fail in far-off keys. Of these three, 1/4 comma temperament is the superior model. All the intervals are closer to true intonation, particularly the major third. A major third from C to E is 386 cents, which corresponds to a frequency ratio of 5/4. Note that

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[24] Barbour, *Tuning and Temperament: A Historical Survey*, 11.
[25] Ibid.
[26] Ibid.
[27] Ibid., 27.
[28] Ibid., 33.
[29] Ibid.

modulation sometimes changes these intervals. C♯ to F is 503 - 76 = 427 cents. This is extremely sharp and shows why meantone temperament allows for only limited modulation.

René Descartes, considered to be the father of modern philosophy, analyzed the way humans perceive sound, and how those perceptions determine dissonance or consonance. In his work, *Compendium musicae,* Descartes begins with eight preliminaries which summarize these ideas. Several of these preliminaries relate directly to temperament. The fourth preliminary states that "an object is perceived more easily by the senses when the difference of the parts is smaller."[30] In terms of temperament, this means that the simplest ratios sound best; the pure major third (5/4) will always sound more consonant than the Pythagorean major third (81/64). Descartes also pointed out that, "Among the sense objects, the most agreeable to the soul is neither that which is perceived most easily nor that which is perceived with the greatest difficulty."[31] This means that in addition to the beauty of simple proportions, there must be some variety. A pure open fifth will sound beautiful, but it can sound bland when it is compared to a complete triad. Descartes observed that there must be a trade-off between simple ratios and interesting complexities. This description of pleasing sound would form a basis for the discussions that followed.

Consider the work of Marin Mersenne, known for his contributions to music and mathematics. In 1636, he published his studies on acoustics in the book *Harmonie universelle*. His idea was that consonance, or "sweetness," is determined theoretically by the simplest ratio.[32] Consequently, the unison is the sweetest and most agreeable sound. However, Mersenne knew that the most simplistic ratio idea is not universally true in practice. As Roger Grant pointed out, "In this scheme, the natural seventh should be more consonant than the fourth compound octave (16:1), which again contradicts conventional knowledge and musical experience."[33] To compensate for this difference between theory and reality, Mersenne made a second stipulation that "the most agreeable

[30] Larry M. Jorgensen, "Descartes on Music: Between the Ancients and the Aestheticians," *British Journal of Aesthetics* 52, no. 4 (October 2012): 409, doi:10.1093/aesthj/ays041.

[31] Ibid.

[32] Roger Mathew Grant, "Ad infinitum: Numbers and Series in Early Modern Music Theory," *Music Theory Spectrum* no. 1 (2013): 67, doi:10.1525/mts.2013.35.1.62.

[33] Ibid.

consonances [are] those produced with the first six integers."[34] With this understanding, aural perception better corresponds with theory. This was a big deal; the mathematical description of consonance influenced the way composers sought to complete the musical circle.

Problems of tuning and temperament affected all keyboard works, including books I and II of J. S. Bach's *Well-Tempered Clavier* (WTC). Yet, Bach seems to have mastered the issue of temperament in these collections by including a piece in every key. How did he do it? According to Thomas Donahue, "The question of temperament and the music of J. S. Bach is complicated…. Bach's music does not seem to be 'supported' by a single temperament."[35] Donahue indicates that history offers varying perspectives on which temperament Bach may have preferred. The only thing known for certain is that Bach preferred his major thirds tuned slightly sharp.[36] Rudolf Rasch writes that until the 1950s, "The WTC was considered to be one of the first examples of what could be done with the tonal system when all twelve semitones were of equal size [ET], so that all keys sounded the same."[37] Conversely, some authors argue that the WTC is best performed with unequal temperament—not technically equal or meantone. Along this line of thought, since Bach composed in different styles for different keys, perhaps he also desired some type of unequal temperament to highlight distinctive elements of each key. As a result, "The tonal relationships are exciting: C major and F major remain the best in tune, E major is the most brilliant key, and there is no harshness anywhere."[38] However, despite much research, Bach's choice of temperament remains shrouded in uncertainty. Although historians will continue to debate the authentic temperament for Bach's music, the ground-breaking truth of the WTC clearly remains today; it is possible to perform an aurally pleasing piece in all twenty-four musical keys on a keyboard instrument.[39] Such an

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[34] Roger Mathew Grant, "Ad infinitum: Numbers and Series in Early Modern Music Theory," *Music Theory Spectrum* no. 1 (2013): 67, doi:10.1525/mts.2013.35.1.62.

[35] Donahue, *A Guide to Musical Temperament*, 117.

[36] John O'Donnell, "Bach's Temperament, Occam's Razor, and the Neidhardt Factor," *Early Music* 34, no. 4 (2006): 632, doi:10.1093/em/cal101.

[37] Rudolf Rasch, *Bach, Handel, Scarlatti: Tercentenary Essays*, ed. Peter Williams, (New York: Cambridge University Press, 2008), 293.

[38] Bradley Lehman, "Bach's Extraordinary Temperament: Our Rosetta Stone—2," *Early Music* 33, no. 2 (2005): 211, doi:10.1093/em/cah067.

[39] Ibid.

accomplishment hints at the complete harmonic freedom composers would soon employ.

As demonstrated by the WTC, it is possible to tune a keyboard instrument so that it sounds good in any key. However, the easiest and simplest way to do this is through the use of ET. Increases in chromaticism throughout the Classical and Romantic periods called for a tuning system which allows for free modulation, particularly enharmonic modulation.[40] While this does not demand that all instruments use ET, "Equal temperament is the best approximation, on an instrument of fixed intonation, to the flexible intonation implied in enharmonic change."[41] Even the great music theorist, Rameau changed his opinion about ET after years of work. Rameau had formerly supported irregular temperaments but decided in 1737 that ET was the better system.[42] While ET has its downsides—mainly the extremely sharp major thirds— it met the needs of composers from the Classical period onward. Rameau serves as just one example that growing harmonic trends of extended chromaticism led to the gradual adoption of equal temperament. The various compositions of the nineteenth and twentieth centuries—from romanticism to atonality—demonstrate the accomplishments of this tuning system. Without ET and the equality it establishes between all pitches, this music could not have been composed. For this reason, after hundreds of years of discussion and hundreds of tuning systems, musicians eventually settled on the ET compromise—equal temperament had nearly limitless potential.

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[40] Duffin, *How Equal Temperament Ruined Harmony*, 36.
[41] L. S. Lloyd, *Intervals, Scales and Temperaments*, 164.
[42] Donahue, *A Guide to Musical Temperament,* 113.

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