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A Viscoelastic Damage Model for Human Cortical Bone

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A VISCOELASTIC DAMAGE MODEL FOR HUMAN CORTICAL BONE

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INTRODUCTION: Skeletal fragility is an important orthopedic concern including the prevention of osteoporosis, long-term stability of prosthetic implants and stress fractures[1]. Damage in human cortical bone has been implicated as a cause of increased fragility[2] and is thought to initiate bone remodeling. It has been demonstrated that bone undergoes viscoelastic deformation in both physiological[3] and elevated strain range[4]. The objective of this study was to develop a constitutive model for human cortical bone based on thermodynamics of irreversible processes, which describes bone's viscoelastic damage behavior.

METHODS: **Model:** A developed formulation was adopted for decomposition of strains caused by time-dependent (viscoelastic) and nonlinear (damage) deformation[5]. It was assumed that time-dependent stress and damage influence the viscoelastic strain through *time dependent effective stress*[6] ($\tilde{\sigma}$). For simplification purposes strains caused by permanent deformation were not included. Considering one-dimensional uniaxial loading case the total viscoelastic strain coupled with damage is given by

$$\varepsilon = \int_0^t J(t-\tau)\frac{\partial\tilde{\sigma}}{\partial\tau}d\tau \qquad (1)$$

where J, the compliance of the virgin material[7] and $\tilde{\sigma}$ are given by

$$J = J_0 + J_1 t^{\kappa} \qquad (2)$$

$$\tilde{\sigma} = \frac{\sigma}{1-\omega(t)} \qquad (3)$$

It has been shown that bone has a threshold driven damage behavior[3], therefore it is reasonable to assume that evolution law for the damage parameter ω ($0<\omega<1$) could be taken in the form[7]

$$\dot{\omega} = \left[\frac{\langle\sigma-\sigma_{TH}\rangle}{C(1-\omega)}\right]^r$$ which upon integration yields $$1-\omega = \left[1-\frac{t}{t_F}\right]^{1/(r+1)}$$ where

$$t_F = \frac{1}{r+1}\left[\frac{C}{\langle\sigma-\sigma_{TH}\rangle}\right]^r \qquad (4)$$

Substituting Eqs. (2-4) into (1) and performing the final integration will yield

$$\varepsilon = J_0\sigma\left[1-\frac{t}{t_F}\right]^{-1/(r+1)} + J_1\sigma t^{\kappa}F\left(1,\frac{1}{r+1},\kappa+1;\frac{t}{t_F}\right) \qquad (5)$$

where F is a hypergeometric function. Material parameters in Equation (5) are obtained from experiments. The numerical simulations using the Eq. (5) were performed using Maple6 (Waterloo Maple) analytical software package.

Uniaxial Tests: Sixteen (n=16) dog-bone shaped specimens (15mm×3mm×1.5mm (gage length×width×thickness) and the radius of curvature of the wasted area (R=5mm)) were machined from a single tibia of a 54-year-old male tibia. The specimens were randomly assigned to three groups (T (n=6), R (n=6) and V (n=4)). Specimens from group T were tested in tension with a loading protocol[4] consisting of ramps with initial rapid load followed by a hold at a desired stress level for 60sec and rapid unload followed by a relaxation for 120sec. A clip-on extensometer (Epsilon) with a gage length of 0.25 inches was attached for strain measurements. Specimens from group R were loaded to stress levels above the threshold value of stress determined from testing of group T and held until failure. And specimens from group V were tested for model verification. All the tests were performed on MTS servohydraulic testing machine.

RESULTS: The results obtained from the specimens from T demonstrated closed hysteretic stress-strain loops thus supporting the assumptions of viscoelastic behavior. As the hold stress level increased those loops became larger indicating more profound dissipation phenomena taking place. Analysis of the instantaneous stiffness obtained from the loading portions of the stress strain curves of the individual ramps, together with the strain rates obtained from the linear portion of the holding stage demonstrated that those values remained constant up until some threshold value of hold stress beyond which a dramatic increase of strain rate and a decrease in the stiffness was observed. The threshold stress (σ_{th} = 75.3 MPa) was determined by fitting the mean on the linear portion of the pooled strain rate versus stress level plot and shifting the mean by a standard deviation. The intersection point between this line and the nonlinear fit of the overall data was identified as the threshold point (Fig1a). The parameters J_0,J_1 and κ were obtained by fitting the load-hold portions of an individual ramps below the threshold using $\varepsilon=J\sigma$ relationship were J is given by Eq. (2) (Fig. 1b). The remaining parameters (r,c) were obtained by fitting time-to-failure versus stress level data, obtained from V using Eq. (4) (Fig. 1c). The final results are summarized in Table 1.

Table 1. Model Parameters

	Mean	Stand. Dev.
J_0 ($\times10^{-6}$MPa^{-1})	37.9034	6.209
J_1 ($\times10^{-6}$MPa^{-1}(sec)$^{-\kappa}$)	10.722	3.596
κ	0.0724	0.016
C	36.617	N/A
r	11.265	N/A

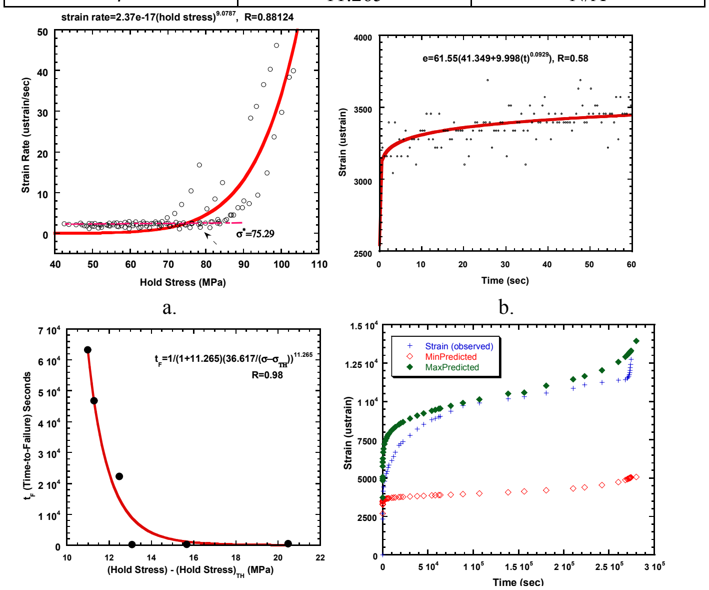

Figure 1. a) Damage evolution, b) typical fit of pre-threshold viscoelastic strain, c) time-to-failure versus (stress-threshold stress), d) predicted verses observed strain history using upper and lower limits of the parameters.

DISCUSSION: Figure 1d demonstrates the ability of the model to predict all three stages of creep observed during the creep to rupture tests performed on the specimens from V. In addition the model was able to predict with moderate accuracy time to failure of the specimens from V. These results suggest that the proposed formulation is able to predict viscoelastic-damaging behavior of specimens. The model could be utilized for predicting deformations in cortical bone and thus could be utilized for studying various orthopedic problems.

REFERENCES:

1. Burr et al., *J. Bone Min. Res.*12:6, 1997. 2. Burr et al., *J. Biomech.* 31:337, 1998. 3. Fondrk et al., *J. Biomech.* 21:623, 1988. 4. Abdeltawab et al., *J. Dam. Mech.,* 7:351, 1998. 5. Lemaitre *A Course on Dam. Mech.,* NY, Springer-Verlag, 1992. 6. Smith et al., *Int. J. Fract.,* 97:301, 1999. 7. Lemaitre and Chaboche *Mech. Solid Mater.,* NY, Cambridge Univ. Press, 1985.

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