Practical Considerations for State Estimation of an Autonomous Vehicle

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Abstract

The Kalman Filter is a widely used algorithm for state estimation and sensor fusion. It can aggregate information from multiple sensors along with a linear state prediction model all while accounting for sources of error probabilistically. In theory, the Kalman filter is an optimal state estimator. In practice, the performance depends on the engineer's ability to quantify a sufficiently accurate linearized prediction model as well as the probabilistic models of measurement and process error. This project is a review of relevant literature and assembly of the pieces of information necessary to implement a practical Kalman filter for the state/localization estimation of an autonomous vehicle. We will focus on the meaning of the various parameters/models, concrete ways of approximating these parameters/models, and what the Kalman filter can and cannot do to make an autonomous car system more robust.

Background

The Kalman filter is a method of digital filtering known as a state estimator. In fact, a Kalman filter is an optimal state estimator for a linear system with normally distributed, zero-mean noise. The Kalman filter has been used in many applications including financial analysis, target tracking, and autonomous vehicle localization. [3] Kalman filters are popular because of their ability to use probabilistic models to efficiently and optimally account for error in real-time systems, provide a prediction, and synthesize information. Kalman filters are often discussed in literature and industry, but we wanted to know what data, models, and parameters would be necessary in order to use one for our autonomous car project.

Benefits

In an autonomous vehicle, a Kalman filter can provide a robust method to use information from multiple sensors to estimate the correct location of the vehicle. Using multiple sensors is advantageous because if one sensor stops providing reliable data (the GPS loses its connection) the other sensor(s), along with the prediction model, can continue to direct the movement of the vehicle. Furthermore, all sensors have error in the measurements, and the Kalman filter helps to account for and correct the error.

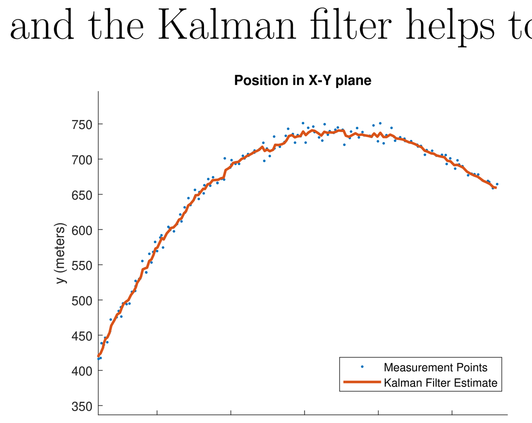

Figure: Planar position of "toy car" simulation, measured and estimated

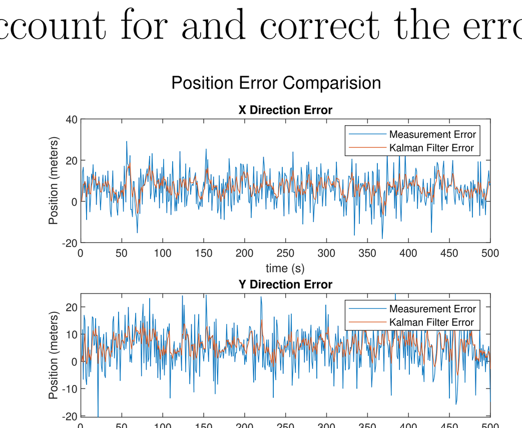

Figure: Error in measurement and estimate for "toy car" simulation

Parts to a Kalman Filter

$$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_k \tag{1}$$

$$P_k^- = AP_{k-1}A^T + Q \tag{2}$$

$$K_k = \frac{P_k^- H^T}{HP_k^- H^T + R} \tag{3}$$

$$\hat{x}_k = \hat{x}_k^- + K(z_k - H\hat{x}_k^-) \tag{4}$$

$$P_k = (I - K_kH)P_k^- \tag{5}$$

These 5 equations govern the Kalman filter. To execute the above equations, the following pieces are needed to provide a complete picture of the system/application.[4]

- Dynamics Model (A): A model using physics to predict where the vehicle will be based on past states.
- Control Input (u_k): Any known inputs to the system.
- Sensor Data (z_k): Measurements of observable facets of the system.
- Measurement Noise Covariance (R): A model of the expected error or noise in the measurements.
- Process Noise Covariance (Q): A model of the expected error or noise in the dynamics model being used to predict the next state of the system.
- Initial Estimate Error Covariance (P_0): An estimate of the error in the initial state of the system.

Assumptions vs Reality

- Error in both measurement and process/model has a Gaussian distribution with zero mean
- The sensors, taken together, give sufficient information to estimate the location of the car

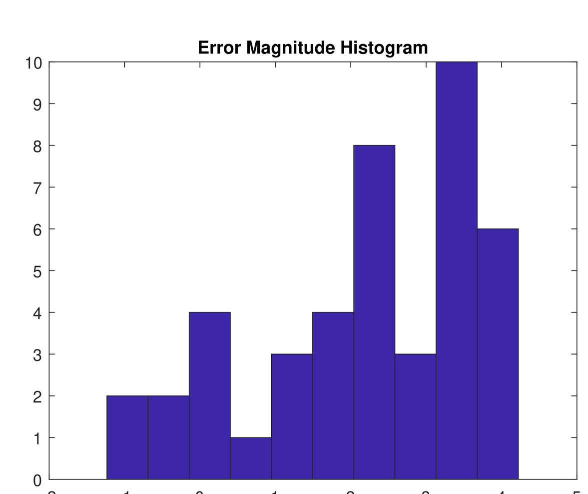

Figure: Histogram of measurement error

Future Investigations

- Further research into how Kalman filters can be used with sensor fusion with redundant sensors
- Test the Kalman filter in the scenario where the car loses GPS signal intermittently
- Investigate the limitations of the Kalman filter with different types of (non-Gaussian) error

Estimation of Covariance Matrices

A covariance matrix of an error vector $E = [e_1 \ldots e_k]^T$ is defined

$$E = \begin{bmatrix} \sigma_{e_1}^2 & \Sigma_{e_1,e_2} & \ldots & \Sigma_{e_1,e_k} \\ \vdots & & \ddots & \\ \Sigma_{e_k,e_1} & \Sigma_{e_k,e_2} & \ldots & \sigma_{e_k}^2 \end{bmatrix},$$

where variance is defined as

$$\sigma_{e_i}^2 = \frac{\sum_{n=1}^{N}(e_i^n - \bar{e}_i)^2}{N-1},$$

and covariance is defined as

$$\Sigma_{e_i,e_j} = \frac{\sum_{n=1}^{N}(e_i^n - \bar{e}_i)(e_j^n - \bar{e}_j)}{N-1},$$

N is the number of samples, and $\bar{e}_i$ represents the mean of those samples.

To estimate the measurement covariance matrices, you need:

- measurements collected from a run
- the true position for the same run

To accomplish this practically, we

- affixed chalk to the golf cart so that a line would be drawn beneath the GPS as it drove
- used surveying equipment to measure points along the chalked line
- interpolated between surveyed points using a cubic spline fit

$Error = GPS - surveyed$ is fed into the equations above to calculate the measurement error covariance matrix, R.

The state covariance P_0:

- Calculated with respect to the state vector itself
- Use a higher value of P_0 when initial measurements should not be trusted.

The process error covariance R:

- There is no direct way to measure the error in our dynamics model
- Letting $Q = 0$ means we assume zero process error
- You could minimize a cost function to estimate Q [1]
- What we did: estimate Q indirectly [2]

Using the control coefficient matrix, the random variance can be projected onto the acceleration standard deviation (σ_a). Then, this result is integrated to produce the continuous time process error covariance matrix as described below

$$Q_c = \int_0^{\Delta t} \sigma_a^2 BB^T dt \tag{6}$$

Acknowledgments

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Results

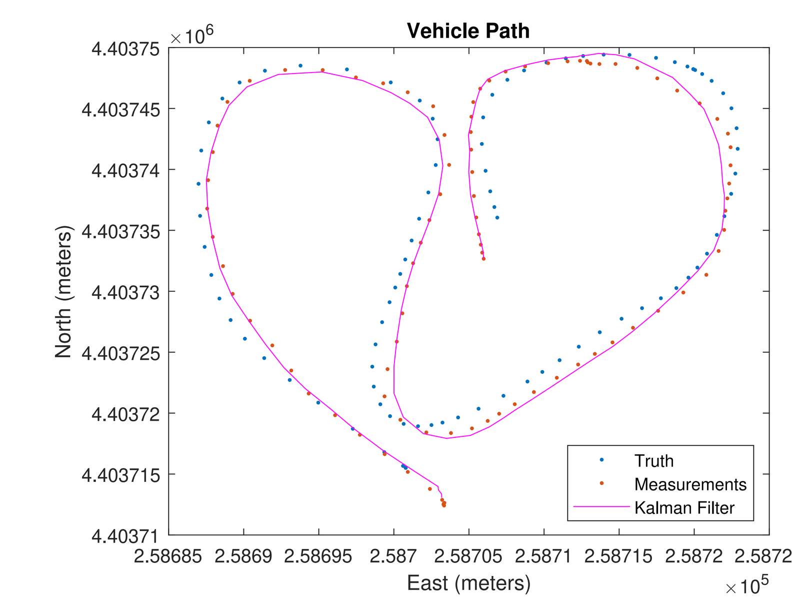

Figure: Planar vehicle path with measurement, estimate, and survey truth

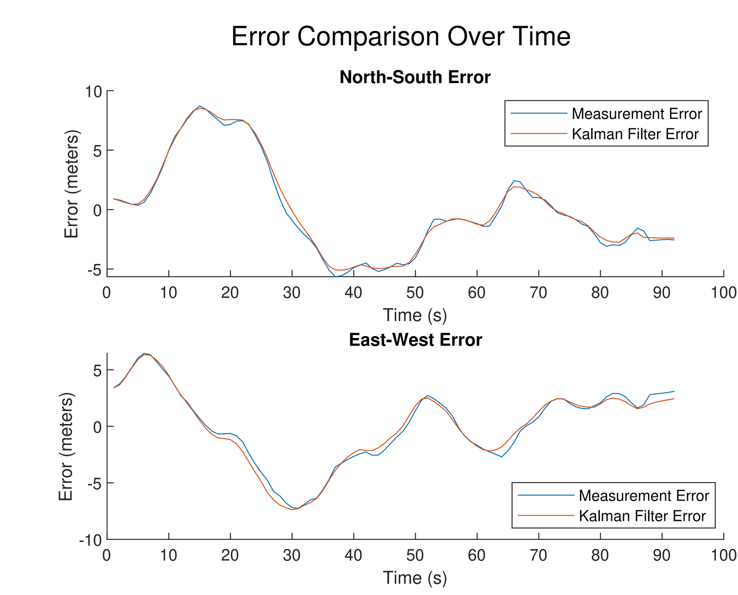

Figure: Error in measurement and estimate

Integral of error in latitude was improved by: 1.57 m-s
Integral of error in longitude was improved by: 1.91 m-s

Conclusion

- We learned the strengths and weaknesses of the Kalman filter for our project
- We present a process for a measurement-based estimate of covariance matrices
- We identified gaps in the application-focused liturature on practical Kalman filter use
- The resulting Kalman filter was tested on simulated and real autonomous car data
- Results show that the Kalman filter made small reductions in error compared to measurement alone
- We identified the next steps to take in order to make a Kalman filter helpful to the autonomous car project

References

[1] Mudambi R. Ananthasayanam. *Tuning of the Kalman Filter Using Constant Gains.* 2018.

[2] Alex Becker. 2018.

[3] Lim Tien Sze Lim Chot Hun Ong Lee Yeng and Koo Voon Chet. *Kalman Filtering and Its Real-Time Applications.* 2015.

[4] Greg Welch and Gary Bishop. *An Introduction to the Kalman Filter.* 1995.