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DISTRIBUTION OF SUPERNOVA REMNANTS IN THE GALAXY

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ABSTRACT
The number of Supernova Remnants (SNRs) observable in the Galaxy is consistent with the number expected to be formed in a Universe that is 7,000 years old. The resulting problem of the "missing Supernova Remnants" is well known and is recognized by astronomers who work in this field.

INTRODUCTION
Supernova Remnants in our own Galaxy and in nearby galaxies are theoretically observable for over one million years before they merge into the inter-stellar background. This theoretical lifespan of over one million years makes a study of the age of SNRs particularly useful for a comparison between a YOUNG Universe Scenario and an OLD Universe Scenario. Since the 'rate of production' of SNRs is now reasonably well determined as being about one every 25 years in the Galaxy, there should be no more than about 280 galactic SNRs if the Universe is only 7,000 years old.

WHAT IS A SUPERNOVA REMNANT?
Strictly speaking, there are a variety of objects that could be described as being remnants of the huge cataclysmic event that we call a Supernova. We know, for example, that pulsars, neutron stars and perhaps black holes, can be formed. However, the term "Supernova Remnant," or SNR, is always taken to refer to the huge cloud of expanding stellar debris that hurtles outwards from the original explosion point at an initial velocity of upwards of 7,000 km.s\(^{-1}\). There are, in general, two main types of SNRs, the 'filled centre' type and the 'shell' type of which the Crab Nebula is a well-known example. The filled centre type of SNR is often called a plerion and is powered partly by the pulsar that energizes its central volume. Plerions comprise about 4% of observed SNRs. It is believed that plerions exhibit some of the features of shell-type when the plerions reach their later stages when they are no longer 'powered' by their central pulsar. (Ref: Lozinskaya [14]) The shell type comprise those objects distinguished by the conspicuous boundaries that are formed by the expanding cloud of stellar debris as they cause a 'snow-plow' effect through the inter-stellar medium (I.S.M.).

Most SNRs that are formed in our Galaxy are initially hidden from our view by inter-stellar dust. These dust particles are of a size that is comparable to the wave-length of visible light and so the dust strongly interferes with our observations. (Ref Zelik [22]) However, as the SNR expands it will start to produce very energetic synchrotron radiation at radio wave-lengths. Photons of these very long wave-lengths are not affected by the inter-stellar dust and most of the galactic SNRs then become easily observable at radio wave-lengths as a result of their huge energies. However, when we look at nearby galaxies such as the large Magellanic cloud and the M33 Galaxy, we are looking outwards from the plane of our Galaxy, and so the problem of the dust particles obscuring our observations at optical wave-lengths does not arise to the same extent. This means that we can readily study SNRs in these nearby galaxies with our optical telescopes as well as radio telescopes. Astronomers have made comprehensive whole-sky surveys of SNRs as well as more detailed studies on individual remnants. Catalogues
have been prepared by astronomers working at observatories such as the Molonglo-Parkes in Australia; the Kitt-Peak in Arizona; the Cambridge 5 Km Array in England; the Jodrell-Bank dish in England; the Effelsburg 100m dish in Germany; the Cornell University Arecibo dish in Puerto Rico and the Very Large Array in Socorro in New Mexico. Useful catalogues are those of Green detailing 174 Galactic SNRs (Ref Green [9]) and an earlier catalogue of Milne (Ref Milne [17]).

A great deal of work has been done, particularly in the last 15 years, on analysing the various parameters of size, age, distance and radiative power of these remnants. Theoretical models have been made which trace the evolution of the remnants over the three main stages of their lifetime. Many surveys of distant galaxies have been made to find the value of \( \tau \), the average rate of occurrence of Supernovae in the various types of galaxies. One of the most striking conclusions of these studies is given by the comments of a team of astronomers writing in a National Research Council handbook. They stated, in the context of a discussion of the Galactic 'Problems of the Decade', "Where have all the remnants gone?" (Ref National Research Council [20]).

This present study provides an analysis of current research on:

1. The value of \( \tau \), the rate of occurrence of Supernovae in the Galaxy.
2. The energy of an SNR.
3. Observational limits.
5. The detailed model of Cloft and McKee.

The resulting data from the above is then used to make a comparison of the expected number of galactic SNRs that should be observable under the 'Young Universe' model and the 'Old Universe' model.

It should be noted that the SNRs in nearby galaxies such as the LMC and M33 are virtually all the same distance away from us, and so we are seeing them all at the same epoch.

Therefore considerations of the time taken for light to travel to us does not effect the calculations of the statistics for the numerical density of those extra-galactic SNRs. (This subject, however, will not be pursued further in the present paper.)

**THE VALUE OF \( \tau \), THE GALACTIC RATE OF OCCURRENCE OF SUPERNOVA**

Supernova are initially so bright in optical wave-lengths that they can be observed in many other galaxies (as long as we look in directions other than our own Galactic plane.) In 1936 a systematic search for supernovae in other galaxies was begun by Fritz Zwicky (Ref Murdin [19] P43). He used the simple technique of photographing a field of galaxies and then comparing the photograph with a reference set of photographs. By this means he personally logged over 120 Supernova events in other galaxies. More sophisticated techniques are now being adopted through the use of computers that can make comparisons between digitally stored sequential images.

Many hundreds of thousands of galaxies have since been compared in this way in order to obtain good data on the frequencies of SN occurrences in galaxies of different types. Currently the best figures we have for the birth-rates in a galaxy similar to our own indicate one supernova event about every 25 years. (Ref Tamman [21]; Murdin [19] P46). \( \therefore \frac{1}{\tau_{\text{Galaxy}}} \approx 25 \text{ years} \)

**THE ENERGY OF AN SNR**

**Initial Energy**

The energy contained by an SNR is quite prodigious. The initial source of this energy is the incredibly fast gravitational collapse of the pre-cursor star in about two seconds. This event produces enough neutrinos to power all the stars in our Galaxy for several years (Ref Henbest [10]) and, for a brief time, will shine with the power of 100 billion stars, thus outshining all of the stars in the galaxy that contains it. This is the reason why Supernovae can be observed in distant galaxies and enables us to obtain the value of \( \tau \) for galaxies of different types.

Clearly a great deal of energy is also imparted to the huge expanding cloud of stellar 'debris.' It is estimated by most theorists that this energy is of the order of \( 10^{51} \) ergs (Ref Duric and Seaquist [8]). To give some idea of the
amount that this represents, it would be sufficient to power 1,000 stars of the same radiative output as our sun for over 8 million years.

### Second Stage Energy

After the relatively short freely-expanding first stage, the SNR enters the long second stage known as the adiabatic or Sedov stage. In this stage it emits ever more strongly at radio frequencies. SNRs are then very prominent objects in all galactic radio surveys as a result of their huge energies. Indeed, if our eyes were sensitive to radio wave-lengths we would be able to see several hundred of these magnificent objects with apparent sizes up to several times the diameter of the moon. Of course, the actual sizes of SNRs can be very large with diameters of old SNRs being theoretically over 300 light years across. This can be compared to our whole solar system which is only about 8 light hours across.

These huge second stage SNRs radiate mainly at radio wave-lengths but some radiate strongly at X-Ray wave-lengths. It was once a mystery as to how an SNR such as the Crab Nebula could radiate at the very high energy synchrotron X-Ray wave-lengths. Synchrotron X-Ray photons are emitted by very fast relativistic electrons spiralling in a magnetic field. However, these high energy electrons have a half-life of only 2 or 3 years as given by the formula.

\[
t = 6 \times 10^{11} B^{-3/2} \nu^{-1/2} \text{ secs (Ref Manchester [16])}
\]

where \(B\) is the component of the magnetic field perpendicular to the electron velocity and \(\nu\) is the frequency of 10^18 Hz for X-Ray emission.

Since the Crab SNR is now 940 years old, the initial energy that produced the X-Ray Radiation would have long since been exhausted. It is thus clear that an on-going process must be powering the Crab SNR in addition to the kinetic energy it was initially given. This on-going energy source is now known to be the Crab pulsar which is situated at the centre of the Crab SNR. The pulsar is losing kinetic rotational energy at the rate of 5 \(\times 10^{36}\) ergs s\(^{-1}\) which is almost exactly the rate required to keep the Crab radiating at its X-Ray level.

However, most SNRs do not rely on pulsars as their main source of radiative energy. The radiative energy of most second-stage SNRs is the synchrotron radiation at radio wave-lengths again caused by electrons travelling in a helix around the extremely long-lived magnetic field of the SNR. The lifetime of these electrons is, however, much longer than that of those emitting X-Rays. Using the formula above, the lifetime of electrons emitting at the 1 GHz level is 31,000 years and at the 10 GHz level is 170,000 years. This is one indication as to why the second-stage SNRs can radiate so strongly and for such a long time.

The total radiative energy expended per second in this second stage is of the order of \(10^{37}\) ergs s\(^{-1}\) (Ref Cioffi and McKee [5]) and it will be shown that most galactic SNRs are easily observable at this radiative level. At a rate of production of \(10^{37}\) ergs it would take over 3 million years for just half of the initial energy of the SNR to be depleted - and this does not take into consideration any additional injection of energy sources such as that supplied by pulsars to the central core.

### Third Stage Energy

In the second stage, the SNR loses very little thermal energy. However, when it enters the third and final stage, thermal radiation predominates. This is known as the isothermal stage and is theorized to last about one to six million years (Ref Ilovaiski and Lequeux [12] P.350). At the end of this stage the SNR is theorized to come to the end of its life-span when it either reaches equilibrium with the ambient pressure at a diameter of \(D_p = 560\) parsec (pc) or collides with similar SNRs (Ref McKee and Ostriker [11]) at a diameter of \(D_{av} = 418\) pc (One parsec = 3.2616 light years).

### OBSERVATIONAL SELECTION EFFECTS

There are three main effects that limit our observation of SNRs at radio wave-lengths.

The first is related to the intensity of the flux density \(S\) (measured in Janskies), received from an individual SNR. \((1\ \text{Jansky} = 10^{26}\ \text{Wm}^{-2}\ \text{Hz}^{-1})\) The flux density received is dependent upon the absolute Surface Brightness \(\Sigma\) of the SNR and also the distance \(d\) to the SNR. \((\Sigma\ \text{is measured in Wm}^{-2}\ \text{Hz}^{-1}\ \text{Sr}^{-1})\) The intensity falls off with the inverse square law and SNRs will not be observable if they are so far away that their radio flux density falls off to a value which is below that at which background confusion sets in. Surveys commonly give the lowest value of \(S\) that is applicable to that particular survey. For example, the pencil-beam survey undertaken by Caswell (Ref Caswell [3]) was said to give a uniform coverage of sources greater than 3 Jy over its fairly limited region of the galactic plane.
The second and third observational limitation involve the difficulties of observing very small or extremely large SNRs respectively. These three effects are important for the conclusions of this paper and are detailed below.

1. **Flux Density Observational Limitation**

Illovaisky and Lequeux (1972, [11] P.174) claim that their table of Galactic SNRs is almost complete for all SNRs with flux densities greater than 10 Jansky.

Other surveys claim an even lower flux density limit. However, the 10 Jy limit will be used in this section. This flux density limit can now be related to the Diameter (D), the Surface Brightness (Σ) and the distance of the SNR from the observer (d) as follows:

\[ \Sigma_{1.4GHz} = 1.19 \times 10^{19} \times S \times \theta^2 \]  

This equation is a standard astrophysical formula that relates the surface brightness to S, the flux density and the angle θ, in minutes of arc, that the SNR subtends at the observer.

\[ \frac{D}{1000d} = \frac{\theta \pi}{180 \times 60} \]  

This is a straightforward trigonometric relationship corrected to the units I am using for D, d and θ (p.c's, kpcs and minutes of arc)

\[ \Sigma = 4 \times 10^{-15} D^{-3.5} \]  

This is a theoretical relationship obtained by Duric and Sequist [8]. They claim that this relationship agrees very well with the observed data.

\[ S \geq 10 \text{ Jy} \]  

These four equations are combined to give the following boundary

\[ D_{\text{min}} = 1160 \times D^{0.2} \]  

which is plotted on the graph (Fig.1). This gives the minimum value of the diameter of an SNR that is observable at a distance of d, using the S ≥ 10 Jy observational limitation.

2. **θ<2° Small SNR Observational Limitation**

This is an observational limiting effect for very small and young SNRs particularly as it effected the Molonglo-Parkes whole sky radio survey (Ref Illovaisky and Lequeux 1972 [11] P175).

By using the relationship

\[ \frac{D}{1000d} = \frac{\theta \pi}{180 \times 60} \]  

and minutes of arc respectively, we obtain

\[ D_{\text{min}} = \frac{0.58d}{\pi} \]  

which gives the minimum 'D' value of a galactic SNR that is observable to us at a distance of d under the θ<2° limitation.

3. **θ>5° Large SNR Observational Limit**

Several authors refer to this effect which applies to those near-by SNRs which have a very large angular extent (e.g. Illovaisky and Lequeux [11] P179 and Caswell [3]). Caswell states that remnants could escape detection "if they are close-by and have a large angular extent"

Illovaisky and Lequeux (op cit) quantify the above as a "practical upper limit of about 5° diameter above which the source, unless strikingly symmetrical, would not be distinguishable from the galactic background."

Substitution in the standard trigonometric relationship of paragraph 2 above gives

\[ D_{\text{max}} = 87d \]  

which is also plotted on the D-d plane.

This relationship

\[ D_{\text{max}} = 87d \]  

provides the maximum value of D for an SNR to be observable at a distance d under this θ>5° observation limit.

These three limiting relationships are plotted on the D-d plane together with those 76 SNRs which have published values for d (Fig.2). These values were obtained from the catalogues of Green [9] and Illovaisky and Lequeux [11]. The resulting plot clearly shows two striking features.

First the values of the observed SNRs fall within the three limiting effects.
Second there is a very clear 'short-fall' of older SNRs with diameters between 60pcs and 260pcs. It should also be noted that the 'D' axis is logarithmic and therefore, the short-fall of large SNRs is actually far more than is immediately apparent when viewing the plot.

The three relationships for \( D_{\text{ring}}, D_{\text{max}} \), and \( D_{\text{max,a}} \) are all given as a function of \( d \) and can be used to calculate the percentage of galactic SNRs that should be observable between any particular range of values of \( D \) (say \( D_1 \) to \( D_2 \)). This important percentage value can be obtained directly from the graph (Fig. 1) or can be evaluated from the various areas of (Fig. 1) by simple integration. It should be noted when performing this calculation that the distance from the earth to the furthest extreme of the Galactic disc is \( \approx 25 \) kpc. The range of 'd' is therefore taken as being from 0 - 25 kpc in order to encompass the whole disc of the Galaxy. Percentage results, evaluated as above, are given in the next section for each of the three main stages of the life-history of a standard SNR.

MODELS OF THE KINEMATIC AND RADIATIVE HISTORY OF A TYPICAL 'SHELL-TYPE' SNR.

The evolutionary track over time of a typical 'shell-type' SNR has been modelled by a number of authors (e.g. Duric and Seaquist [8]; Ilovaisky and Lequeux Paper I [11]; Cox [6]; Chevalier [2]; McKee and Ostriker [18].) These models have been progressively refined to a high level of sophistication and confidence. To give one example, the onset of the second stage, or the 'blast-wave' stage, has been variously given as occurring at any time from about 60 years to about 600 years after the initial explosion. Recently the researchers Band and Liang (Ref Band and Liang [1] P69) at the Livermore National Laboratory, used a Cray supercomputer to complete several numerical simulations of this transition phase. The interactions involved during this relatively brief period are just too complex to allow for a concise and accurate Mathematical analysis. The numerical simulations, however, showed that the onset of the shock wave for a 'standard' SNR occurs at about 317 years. (A 'standard' SNR is taken by most authors as having an initial energy totalling \( 10^{51} \) ergs and as expanding into the I.S.M. with an ambient gas density \( \rho = 10^{-24} \) gm. cm\(^{-3} \).

For the purpose of this paper, it is not necessary to detail all of the above models since they give broadly similar dynamic and radiative life-history tracks for a standard SNR. We will follow the detailed model of Cioffi and McKee of the University of California, Berkeley, for their accurate treatment of the important Second and Third stages. Cioffi and McKee claim to have achieved a relatively simple expression for accurate kinematics of SNR expansion and they imply an accuracy of \( \leq 5\% \) for the overall kinematics.

DETAILED KINEMATIC MODELS

First Stage of Expansion

Using a Cray computer, Band and Liang performed numerical simulations of the initial expansion of a supernova into the surrounding medium and followed the expansion towards the adiabatic blast wave stage (stage 2). They assume the expansion of a 1.4M star with initial expansion energy of \( 10^{51} \) ergs and expanding into a medium of \( \rho = 10^{-24} \) gr cm\(^{-3} \) (Ref Band and Liang [1] P71). Other initial assumptions would result in, for example, a range of sizes of SNRs of a given age scattered about the 'mean' of the standard SNR. However, this would not effect the overall numerical range of values of \( D \) that is being discussed in this paper. (The value of \( \tau \) is by far the most important determining factor for the numerical density of galactic SNRs and this value is independent of the values of these initial assumptions.)

The results of this simulation showed that the blast wave (that signifies the end of the first stage) formed at a time of 317 years. The value of the diameter of the SNR at 317 years can be obtained from the following equation (after Duric and Seaquist (1983))

\[
D(t) = 2.3 \ E_o^{0.6} \rho_o^{0.4} \tau
\]

where \( E_o \) is the energy input from the explosion = \( 10^{51} \) ergs and \( \rho_o \) is the ambient gas density = \( 10^{-24} \) gr. cm\(^{-3} \). This gives \( D_{317,\text{yr}} = 7 \) pc.

It is now very easy to calculate the total number of Stage One SNRs that should be extant in the Galaxy. \( \tau \) for the Galaxy \( \approx 25 \) years, therefore there should be \( \approx 12 \) First Stage SNRs.

Using the observational limitation formulae obtained in the last section, the actual number that should be observed will be 19% of 12 = 2 observable SNRs.
In Summary:

Total # of First Stage SNRs expected to be observed under an 'Old Universe' Scenario = 2

Total # of First Stage SNRs expected to be observed under a 'Young Universe' Scenario = 2

Actual # of First Stage SNRs observed with a diameter range from 0 - 7pcs = 5

Second Stage of Expansion

A kinematic model for the evolution of an SNR was developed by D.F. Cioffi and C.F. McKee of the University of California, Berkeley (Ref Cioffi and McKee [5] P435.) They model the Second Stage after the classic Sedov-Taylor solution together with a 'pressure driven snow-plow' solution and with a cooling function \( \propto T^2 \). The authors claim that a comparison between their analytical model and a numerical solution agrees to within 20% as long as \( t < t_{pds} \), where \( t_{pds} \) is the onset of the 'pressure driven snow-plow' which is calculated as being 50,000 years. This means that they claim that level of accuracy up to an SNR age of at least 1,000,000 yrs.

They found that the total luminosity of the SNR actually increases in value from about \( 10^{36} \) ergs s\(^{-1}\) at a theoretical age of 10,000 years up to about \( 1.9 \times 10^{38} \) ergs s\(^{-1}\) at a theoretical age of 120,000 years. This increase in overall luminosity, of course, will result in the SNRs maintaining a high level of observability throughout the Second Stage.

The diameter of the SNRs over this period of time will increase from 7pc to 104 according to the following expression of Cioffi and McKee

\[
D = 14.0 \times \left[ \frac{(E_{51})^{\frac{2}{7}}}{n_0^{\frac{3}{7}} \cdot \zeta_m^{\frac{1}{7}}} \right] \times \left[ \frac{4}{3} \cdot \frac{t}{t_{pds}} \cdot \left( \frac{10}{3} \right)^{\frac{3}{10}} \right]
\]

Using the given values of Cioffi and McKee as follows: \( E_{51} = \) initial energy in units of \( 10^{51} \) erg, \( n_0 = \) the hydrogen density of the interstellar medium, \( = 0.1 \text{cm}^{-3} \) and \( \zeta_m = \) the metallicity \( = 1 \) for cosmic abundances.

Note that this relation only applies for values of \( t \) after \( t_{pds} \).

At the beginning of the second stage \( t = 317 \) yrs and \( D = 7 \) pc.

At the end of the second stage \( t = 120,000 \) yrs and \( D = 104 \) pc.

Once again using \( t = 25 \) years gives a total number of 4,800 extant, Second Stage galactic SNRs. Of these 47% will be observable, or 2,256 as calculated by the method of the last section on observational limitations.

In summary:

Total # of Second Stage SNRs expected to be observed under an 'Old Universe' Scenario = 2,256

Total # of Second Stage SNRs expected to be observed under a 7,000 year old Universe with \( t = 25 \) = 268

Actual # of Second Stage SNRs observed with a diameter range from 6pc to 106pc = 200

Third Stage of Expansion

Cioffi and McKee assume a \( T^{-1/2} \) thermal cooling rate for their Third Stage model.

They find that the Third Stage starts at a theoretical age of 120,000 years with a diameter of 104pc and a total luminosity of \( 1.9 \times 10^{36} \) erg s\(^{-1}\). The third stage then continues for 1,000,000 years at which time the luminosity has dropped to \( 6 \times 10^{36} \) erg s\(^{-1}\) when the diameter is 200pc.

Once again, using \( t = 25 \) years, we would expect a total of 35,200 Third Stage SNRs to be extant in the Galaxy of which approx 14.3% or 5,033 should be observable.
In summary:

Total # of Third Stage SNRs expected to be observed under an 'Old Universe' Scenario = 5,033
Total # of Third Stage SNRs expected to be observed under a ’7,000 year old Universe’ Scenario = 0
Actual # of Third Stage SNRs observed with a diameter range from 104pc to 200pc = 0

CONCLUSION

A Young Universe Model provides a very different prediction from an Old Universe Model as to the expected number of observable SNRs and their range of diameters.

A Young Universe Model fits the data. There is a short fall, however, of over 7,000 galactic SNRs based upon the ‘Old Universe Model.’ A number of astronomers, in the context of trying to find solutions to the short-fall, have commented on the situation as follows:

“Major questions about these objects that should be addressed in the coming decade are: Where have all the remnants gone?”

National Research Council Astronomy Survey Committee
Ref: National Research Council [20]

“Another surprise is how rare Crab Nebula type SNRs are”.
Dr Malcolm Longair, Royal Observatory, U.K.

Ref: The New Physics

“Why have the large number of expected remnants not been detected?”

Clark and Caswell
Ref: Clark and Caswell [4]

“The mystery of the missing remnants”

Clark and Caswell
Ref: Clark and Caswell [4]

“There are about 340 SNRs in the L.M.C. which lie above the limit of detection of the Mills Cross (radio-telescope)...these SNRs should also be visible”.

Matthewson and Clarke
Ref Matthewson and Clarke [15]

(The above comment was made based upon the expectation of the number of SNRs that should be observable under an ‘Old Universe’ model. The survey actually found a total of 9 instead of the 340 that were expected)

“The final example is the SNR population of the Large Magellanic Cloud. The observations have caused considerable surprise and loss of confidence”.

Dr D. Cox
Ref: Cox [7]

Since the time of the above two quotes, more sensitive surveys have discovered a further 20 SNRs in the L.M.C. Since the L.M.C. is approximately 1/10 the size of the Galaxy, then $t_{LMC} = 250$. Therefore, the Creationist Model would provide for a total of 24 SNRs in the LMC which again fits the data.
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The three major observational limitations plotted on the D-d plane.

Large SNR's not observable in this region for $\theta > 5''$ limitation

SNR's not observable in this region for $S < 10J_\nu$ limitation

SNR's observable in this region

$D = 8\pi d$

$D = 1162\pi d^{1/3}$

$D = 58d$

Small SNR's not observable in this region for $\theta < 2'$ limitation

[FIG 1]
Galactic SNR's with published values of 'D' (diam in pc) and 'd' (distance in Kpc)

[FIG 2]