THE BATTLE BETWEEN IMPECCABLE INTONATION AND COMPLETE CHROMATICISM
While equal temperament is now universally hailed as the standard tuning system, it is not perfect.

Rather, it represents a compromise designed to best accommodate the needs of tonal music since the Baroque Era.
# Basic Intervals

<table>
<thead>
<tr>
<th>Interval</th>
<th>Freq. Ratio</th>
<th>Decimal</th>
<th>Cents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect Octave</td>
<td>2:1</td>
<td>2.00</td>
<td>1200</td>
</tr>
<tr>
<td>Perfect Fifth</td>
<td>3:2</td>
<td>1.50</td>
<td>702</td>
</tr>
<tr>
<td>Perfect Fourth</td>
<td>4:3</td>
<td>1.33…</td>
<td>498</td>
</tr>
<tr>
<td>Major Third</td>
<td>5:4</td>
<td>1.25</td>
<td>386</td>
</tr>
<tr>
<td>Minor Third</td>
<td>6:5</td>
<td>1.20</td>
<td>316</td>
</tr>
<tr>
<td>Major Sixth</td>
<td>5:3</td>
<td>1.66…</td>
<td>884</td>
</tr>
</tbody>
</table>

Cent value = 3986 × log (frequency ratio)
Defined via fifths: Frequency Ratio \[ \frac{3}{2} \times \frac{3}{2} \times \cdots \times \frac{3}{2} \times \frac{3}{2} = \left(\frac{3}{2}\right)^{12} \]

Defined via octaves: Frequency Ratio \[ \frac{2}{1} \times \frac{2}{1} \times \cdots \times \frac{2}{1} \times \frac{2}{1} = 2^7 \]
The Problem

- \( \left( \frac{3}{2} \right)^{12} = \frac{531441}{4096} \approx 129.75 \)
- \( 2^7 = 128 \)
- This difference is equivalent to 24 cents
  - Pythagorean Comma
- “In order for the twelve pitches generated through the proportion 3:2 to complete a path from do to do, the circle has somehow to be adjusted, ‘rounded off.’”
Various Solutions

- **Intonation**
  - Just Intonation
  - Pythagorean Intonation

- **Temperament**
  - Mean-tone Temperament
  - Equal Temperament
Pythagorean and Just Intonation

Pythagorean circle of fifths

Just intonation circle of fifths

Wolf fifth

3-2\(^{6}/5^{3}\)
Equal Temperament

- Divide up the Pythagorean comma equally
- Define octave to be 2:1
  - Break up octave into 12 equal semitones (100 cents each)
  
  \[ 2^{\frac{1}{12}} \approx 1.059463 \]
  
  - One half step (A 440 to A# 466 Hz)
  
  \[ 2^{\frac{7}{12}} \approx 1.498 \]
  
  - Seven half steps (fifth)
Meantone Temperament

- Flatten the fifth
  - 1/4 comma
  - Works very well in closely related keys
  - Breaks apart in keys that are further off

<table>
<thead>
<tr>
<th>C₀</th>
<th>C# - 7/4</th>
<th>D⁻¹/₂</th>
<th>Eb⁻³/₄</th>
<th>E⁻¹</th>
<th>F⁺¹/₄</th>
<th>F#⁻³/₂</th>
<th>G⁻¹/₄</th>
<th>G#⁻²</th>
<th>A⁻³/₄</th>
<th>Bb⁺¹/₂</th>
<th>B⁻⁵/₄</th>
<th>C₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>76</td>
<td>193</td>
<td>310</td>
<td>386</td>
<td>503</td>
<td>579</td>
<td>697</td>
<td>773</td>
<td>890</td>
<td>1007</td>
<td>1083</td>
<td>1200</td>
</tr>
</tbody>
</table>
Listening Examples

- **Tone Generator**
  - Perfect fifth (660 Hz)
  - Wolf-fifth (651 Hz)
  - Meantone fifth (658 Hz)
  - Equal-tempered fifth (659.26 Hz)
  - Equal-tempered third (554 Hz)
History of Temperament

- Pythagoras – 550 B.C.
- Francisco Salinas – 1513-1590
- Rene Descartes – 1596-1650
  - Trade-off between simple ratios and interesting complexities
History of Temperament

- J.S. Bach – 1685-1750
  - Complicated
  - Equal or unequal?
    - Different styles for different keys?
  - Scholars remain unsure
  - Regardless of exactly which temperament Bach used, the point of the WTC clearly remains.
    - It is possible to perform in all 24 musical keys on a keyboard instrument, and to sound good doing it.
Thesis

- While equal temperament is now universally hailed as the standard tuning system, it is not perfect.
- Rather, it represents a compromise designed to best accommodate the needs of tonal music since the Baroque Era.
Questions??